Numerical solution of Riccati equation

Given a matrix differential equation, i.e., Riccati equation as

There is no analytic solution in general. Hence we may use numerical methods to get a solution. Of course there are theoretic algorithms to solve it, however **Already** there are several software package to solve such kinds of matrix differential equations.

Lucky…

Matlab provides us. Another one “python”. Or U may find “C++”.

In this course, I would like to use Matlab.

There are two kinds of Riccati equations. One is continuous, the other one is discrete. And at each case, Dynamic case and Algebraic case.

* Type of Riccati

1. Continuous Riccati equation

* Dynamic case
* Steady state case (CARE- Continuous Algebraic Riccati Equation)

It is provided by Matlab function as “care”

1. Discrete case

* Dynamic case
* Steady state case (DARE- Discrete Algebraic Riccati Equation)

It is provided by Matlab function as “dare”

* The difficulty of Continuous Riccati differential equation.

Even if U may apply a numerical method, in case of dynamic Riccati equations are a little bit complicated. Since symmetric matrix has differential equations, and they are co-related.

* Hence, I will consider only scalar continuous differential Riccati equation. Sorry.
* Differential equation in matlab ;

Matlab command “ode45”

1. First consider a first order differential equation.

1.1 linear homogeneous

%%%%% Week\_7.m

%% linear homegeneous

clear all; clc; clf

tspan = [ 0 5]; % domain

x0 = 1; % initial point

[t,x] = ode45(@(t,x) -x, tspan,x0); % ode45 function.

plot(t,x); grid on

Easy! The function “ode45” stands for oddinary differential equation. ‘45’ means the algorithm version( Am I right?)

If you help ode45 in matlab commnad line, U may get an expalnations by matlab. Let me explain a little bit.

“ode45(@(t,x) -x, tspan,x0)”

* @(t,x) : in matlab we call as an anonymous function definition. Why anonymous? U may think.
* In our case ode45(@(t,x) -x,...) , some function is defined as
* The interval of the function is defined as tspan
* The initial condition is

It is easy..

Let’s in-homogeneous

* 1. Linear inhomogeneous

Change only “ode45”.

%% linear in-homogeneous

[t,x] = ode45(@(t,x) -x + 2, tspan,x0); % ode45 function.

* 1. Non-linear

Let’s consider

Change only “ode45” as

%% non-linear converge

[t,x] = ode45(@(t,x) -x - x^2 +1, tspan,x0); % ode45 function.

Good… sometimes it is divergent, as

Change the domain to see the trajectory.

* 1. Scalar Riccati equation Example 3.6

%% (3-142) Riccati equation

clear all; clc; clf

tspan = [ 0 1]; % domian

s0 = 0; % initial point

alpha = 0.5;

k = 150;

r = [100 1000 10000];

for i = 1:3

[t s] = ode45(@(t,s) 1 - k^2/r(i) \* s^2 - 2\*alpha\*s, tspan,s0); % ode45 function

plot(1-t,s); grid on; hold on

end

s0 = [0 0.19 0.5]; % initial point

r = 1000;

for i = 1:3

[t s] = ode45(@(t,s) 1 - k^2/r \* s^2 - 2\*alpha\*s, tspan,s0(i)); % ode45 function

plot(1-t,s); grid on;

end

hold off

title('Fig.3.7 p(t) for various values of rho and phi')

* 1. Matrix Riccati Equation

Well, there are few open software to solve differential matrix Riccati equation…. I will skip.